DOCUMENT RESUME

ED 121 627 SE 020 803

AUTHOR O'Reilly, Robert R.
TITLE Attitudes Concerning the Nature of Mathematics, Mathematics Teaching and Learning: A Partial Replication of the I. E. A. Study.
PUB DATE 22 Apr 76
NOTE 38p.; Paper presented at the Annual Meeting of the National Council of Teachers of Mathematics (54th, Atlanta, Georgia, April 22, 1976); Occasional marginal legibility in charts due to broken type

EDRS PRICE MF-$0.83 HC-$2.06 Plus Postage
DESCRIPTORS Achievement; *Attitudes; Instruction; *Mathematics Education; *Questioning Techniques; *Research; Secondary Education; *Secondary School Mathematics; Student Attitudes
IDENTIFIERS Research Reports

ABSTRACT

The purpose of the paper is to examine some attitudes high school students have of mathematics. This is a partial replication of the Project for the International Evaluation of Educational Achievement in Mathematics (IEA). The principal variables under study were mathematics achievement as measured by the Stanford Achievement Test; student views of teaching and learning; student belief about the nature of mathematics as an open system; and student attitudes. Specific attitudes investigated were beliefs that (1) mathematics is important, (2) mathematics is difficult, (3) school and school learning is satisfying, and (4) man can control his social and physical environment. Data were collected from 48 9th- and 10th-grade classes in 12 schools in Ontario. The results indicate that inquiry methods of teaching are related to achievement, particularly for more advanced students, that achievement is related to the attitude, and that mathematics is important, but difficult to learn. For students in advanced classes, achievement is related to the belief that mathematics is an open process. (Author/SD)

****
* Documents acquired by ERIC include many informal unpublished *
* materials not available from other sources. ERIC makes every effort *
* to obtain the best copy available. Nevertheless, items of marginal *
* reproducibility are often encountered and this affects the quality *
* of the microfiche and hardcopy reproductions ERIC makes available *
* via the ERIC Document Reproduction Service (EDRS). EDRS is not *
* responsible for the quality of the original document. Reproductions *
* supplied by EDRS are the best that can be made from the original. *
****

U S DEPARTMENT OF HEALTH,
EDUCATION & WELFARE
NATIONAL INSTITUTE OF
EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

ED121627

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

FIFTY-FOURTH ANNUAL MEETING

ATTITUDES CONCERNING THE NATURE OF MATHEMATICS, MATHEMATICS TEACHING AND LEARNING: A PARTIAL REPLICATION OF THE I.E.A. STUDY

PRESENTED BY

ROBERT R. O'REILLY
UNIVERSITY OF OTTAWA

HILTON HOTEL
ATLANTA, GEORGIA

APRIL 22, 1976

SE 020 803

ATTITUDES CONCERNING THE NATURE OF MATHEMATICS, MATHEMATICS TEACHING AND LEARNING: A PARTIAL REPLICATION OF THE I.E.A. STUDY[1]

Although attitudes of both teachers and pupils are thought to be important factors in the learning of pupils, little research can be found on these topics. The typical study in this area is limited to studying the interests of students or their liking for the subject, the class or the teacher (Dutton, 1956 and 1968). Most reviews of research on the teaching of mathematics ignore this topic entirely, (Henderson, 1963; Rombey, 1969), although one exception is the review by Aiken (1970).

The purpose of this project is to examine the attitudes and beliefs of secondary school mathematics students, specifically:
(a) their views of the methods teachers of mathematics employ,
(b) the beliefs of students about the nature of mathematics, and
(c) their attitudes about the importance and utility of mathematics.
The scales used to measure these attitudes were developed by the International Project for the Evaluation of Educational Achievement (IEA). The scales are described by R.M. Wolf in the report edited by Husén (1967).

These attitudes are examined in relation to student characteristics, classroom environment and student achievement. Data are drawn from a major study which included these variables (O'Reilly and Garland, 1974)[2]. This analysis provides an extension and one of the few replications of the IEA study.

---

1 The assistance of Max Trask in analyzing the data used in this paper is acknowledged.

2 Data for this paper is from the research project, Paradigm for Evaluation in the High School, University of Ottawa, 1974. The project was partially supported by a grant-in-aid to Educational Research by the Ontario Ministry of Education.

Framework

Although attitudes of students towards mathematics may influence their achievement, it is probable that the nature of the attitude-achievement relationship changes during the student's school career. Research by Anttonen (1968) and Dutton (1968) indicate that attitudes do change from the elementary to the senior secondary level. Neale (1969) suggested that there is a reciprocal influence between attitude and achievement.

Although the review by Aiken indicates inconsistent results of studies of the attitude-achievement relationship, most studies indicated a low correlation between the two variables (Aiken, 1970).

The data from the IEA study indicate no clear relationships between achievement in mathematics and the attides that mathematics is difficult or important. Only one sub-group (non-mathematics majors in final secondary year) yielded a positive correlation ($r = .13$) for the latter variable (Husén, Vol. II, p. 148). Neale's analysis of the results of the American students who participated in the IEA study also showed no relationship (Neale, 1969).

A common assumption of many educators is that discovery, or heuristic, methods of instruction lead to higher levels of achievement.

A recent essay on this topic by Hughes (1974) traces this movement over the past century. He points out that heuristic techniques fall in the domain of mathematical thinking which is psychological rather than in the domain of mathematical thought which is logical.

Jerome Bruner, in an analysis of student thinking in mathematics, points out that young students must begin with sensory objects and then gradually reduce the specific properties until an abstract level is attained.

> However, once abstraction is achieved, the learner becomes free in a certain measure of the surface appearance of things, he nonetheless continues to rely upon the stock of imagery he has built en route to abstract mastery. (Bruner and Kenney, 1973, p. 436).

Thus the success of heuristics, for the student, is the stock of early concrete experiences and subsequent sets of imagery which permits the student to work at the level of heuristic.

The IEA study reports positive correlations between the view that teachers use "discovery techniques" and achievement for the sample of 13 years old, but small and negative correlations for 18 years old students (Husén, vol. II, p. 145-149). Husén suggests that the latter result may be the result, either of conservative methods of teaching, or of the nature of the examination system. (Husén, vol. II, p. 299).

The value of replicating the test of this hypothesis at this time in the Province of Ontario is suggested by two facts. Modern mathematics programs, with emphasis on "discovery methods" were mandated by a central authority throughout the province beginning in 1962. Teachers in the Province are now thoroughly trained in the program; pupils have known only this type of program. Secondly, there are no important centrally-imposed examinations for the Ontario secondary school students who participated in this study. Thus Husén's explanations would not be applicable in this situation.

A surprising result of the IEA calls for a serious re-examination of the nature of mathematics as viewed by secondary students. Generally there was a negative correlation between achievement and the belief that mathematics is an open system. The association of scores was stronger for older students. Students who were most interested in mathematics tended to view it as a fixed, formal closed system.

Since these results are contrary t the expectations of most authors of modern mathematics programs, the hypothesis should be re-examined with a population which has had extensive experience in the modern curriculum.

Design of the Study

The unit of analysis in this study is the classroom, and all scores are average classroom scores. Forty-eight classrooms from 12 secondary schools from four boards of education in Eastern Ontario constituted the sample. To fulfill certain requirements of the major project, a combination of random and judgmental sampling was employed.

The forty-eight classes were equally divided between the grades 9 and 10 levels (or equivalents) and between advanced academically-oriented classes and general or terminal classes. A total of 1100 students participated.

Data Collection

Data were collected from students with the following instruments:

1. Stanford Achievement Test (SAT): Mathematics Form W.
2. Level of parents' education.
3. Learning Environment Inventory (LEI).
4. School and College Ability (SCAT) Test, Series II.
5. School Mathematics Inventory

Questionnaires and tests were administered in two sitting with each of the 48 classes. One sitting involved the administration of the Stanford Achievement Test-Mathematics - Part A, and the appropiate forms of the SCAT-Series II. The Stanford Achievement test was administered to 2/3 of the class while the SCAT was administered to the remaining one third. In the other sitting, the Learning Environment Inventory and the parents' education Inventory were administered to one half of the class while the school mathematics Inventory was administered to the remainder of the class. Males and females were proportionally represented in each segment of data collection.

The order of administering sets of instruments in the two settings was alternated to eliminate order effects.

The Learning Environment Inventory was developed by Walberg and Anderson for the evaluation of the Harvard Project Physics. The instrument consists of 105 items describing typical school classes. Each student expresses his agreement or disagreement on a four point scale. The items are grouped into 15 scales.

The intra-class correlations for class groups range from .31 to .92 for each scale. The LEI was successfully utilized in a variety of experimental and correlational studies to relate classroom climate to teaching methods and to achievement. (Anderson and Walberg, 1974).

The School Mathematics Inventory was developed by the Council of the International Project for the Evaluating of Educational Achievement (IEA), which conducted a twelve nation cross-national study of mathematics achievement in 1963-64.

Their sample consisted of four panels: (a) thirteen years old, (b) classes in which most students were 13 years old, (c) mathematics students in terminal secondary year, (d) non-mathematics students in terminal secondary year. These groups are referred to as populations 1a, 1b, 3a and 3b respectively.

The first variable defines the teaching methods employed. Views about mathematics teaching ranges from a view that a teaching-learning situation which stimulates students by means of an inquiry approach to a view that teachers employ mechanistic, formalistic modes with great emphasis on rote memorization. The possible range of scores is from 0 to 22. The items are listed in Table 1.

TABLE 1 ABOUT HERE

The second descriptive variable defines the climate of the school and school learning. Views about school learning describes situations which vary from a teacher-centered, authoritarian based program to one which promotes inquiry methods, pupil activity and discovery. The possible range of scores is from 0 to 22. The items are listed in Table 2.

TABLE 2 ABOUT HERE

The IEA analysis assigned to each student the mean score achieved by his class on these first two scales.

TABLE 1. VIEWS ABOUT MATHEMATICS TEACHING

ITEMS
1. My mathematics teacher shows us different ways of solving the same problem.
2. My mathematics teacher does not like pupils to ask questions after he has given an explanation. (D)
3. My mathematics teacher wants pupils to solve problems only by the procedures he teaches. (D)
4. My mathematics teacher expects us to learn how to solve problem by ourselves but helps when we have difficulties.
5. In my mathematics class, pupils who have original ideas get better marks than do pupils who are most careful and neat in their work.
6. My mathematics teacher requires the pupils not only to master the steps in solving problems but also to understand the reasoning involved.
7. My mathematics teacher encourages us to try to find several different methods for solving particular problems
8. My mathematics course requires more thinking about the methods of solving problems than memorization of rules and formulae.
9. My mathematics teacher wants us to discover mathematical principles and ideas for ourselves.
10. My mathematics teacher explains the basic ideas; we are expected to develop the methods of solution for ourselves.
11. Most of the problems my mathematics teacher assigns are to give us practice in using a particular rule or formula. (D)

TABLE 2. VIEWS ABOUT SCHOOL AND SCHOOL LEARNING.

ITEMS
1. Most school work is the memorizing of information. (D)
2. In our school we get a great deal of practice and drill until we are almost perfect in our learning. (D)
3. The pupils spend most of their class time listening to the teachers and taking notes. (D)
4. Our teachers want us to do most of our learning from the textbook which is used in the course. (D)
5. We are expected to learn and discover many ideas for ourselves.
6. We are expected to develop a thorough understanding of ideas and not just to memorize information.
7. Our teachers believe in strict discipline and each pupil does exactly what he is told to do. (D)
8. Pupils are encouraged to devise their own projects or experiments in order to learn on their own.
9. Most of our classroom work is listening to the teacher. (D)
10. We do not use just one textbook for most of our subjects. Various sources and books from which we can learn are suggested to us.
11. Much of our classroom work is discussing ideas and problems with the teacher and the other pupils.

A description of the five attitude scales follows. The underlying continuum for attitudes toward mathematics as a process ranged from a view that mathematics is a fixed, formal system governed by rigid and unchanging rules which a student had to master to a view that mathematics was a subject that was still in a process of development. The eight items are shown in Table 3.

TABLE 3 ABOUT HERE

The underlying continuum for attitudes about the difficulties of learning mathematics range from a view that only an elitist few could learn it to a view that mathematics is not difficult. The seven items are shown in Table 4.

TABLE 4 ABOUT HERE

The scale measuring attitudes toward school and school learning suggests a range from dislike for schooling and a desire to leave it to a genuine liking for school work. The eleven items are shown in Table 5.

TABLE 5 ABOUT HERE

The underlying continuum for the scale measuring attitudes toward the place of mathematics in society range from a view that mathematics is neither essential nor useful to a view that mathematical knowledge is important not only in terms of obtaining a good job but also in terms of national development. The possible range of scores is from 0 to 16. The items are listed in Table 6.

TABLE 3. ATTITUDES TOWARD MATHEMATICS AS A PROCESS.

ITEMS
1. In mathematics there is always a rule to follow in solving problems. (D)
2. The most important reason for studying arithmetic and secondary school mathematics is that they help people to take care of their own financial affairs. (D)
3. Mathematics helps one to think according to strict rules. (D)
4. Almost all of present-day mathematics was known at least a century ago. (D)
5. Mathematics is a very good field for creative prople to enter.
6. There is little place for originality in mathematics. (D)
7. Mathematics will change rapidly in the near future.
8. In the study of mathematics, if the pupil misses a few lessons it is difficult to catch up. (D)

TABLE 4. ATTITUDES ABOUT THE DIFFICULTIES OF LEARNING MATHEMATICS.

ITEMS
1. Anyone can learn mathematics.
2. Very few people can learn mathematics. (D)
3. Almost anyone can learn mathematics if he is willing to study.
4. Any person of average intelligence can learn to understand a good deal of mathematics.
5. Even complex mathematics can be made understandable and useful to every high school pupil.
6. Almost all pupils can learn complex mathematics if it is properly taught.
7. Only people with a very special talent can learn mathematics. (D)

TABLE 5. ATTITUDES TOWARD THE PLACE OF MATHEMATICS IN SOCIETY.

ITEMS
1. More of the most able people should be encouraged to become mathematicians and mathematics teachers.
2. Outside of science and engineering, there is little need for mathematics (algebra, geometry, etc.) in most jobs. (D)
3. Mathematics is of great importance to a country's development.
4. Mathematics (algebra, geometry, etc.) is not useful for the problems of everyday life. (D)
5. A thorough knowledge of advanced mathematics is the key to an understanding of our world in the twentieth century.
6. It is important to know mathematics (algebra, geometry, etc.) in order to get a good job.
7. Unless one is planning to become a mathematician or scientist, the study of advanced mathematics is not very important. (D)
8. In the near future most jobs will require a knowledge of advanced mathematics.

TABLE 6 ABOUT HERE

The scale measuring attitudes toward man and his environment assesses the student's view of the relationship between man and his environment. According to Husén, low scores suggest the belief that man is at the mercy of his social and physical environment. An examination of the items (see Table 7) suggests that the scale also measures the student's acceptance of the rationalist assumptions that education, medicine, engineering and scientific research will solve most of the world's problems. The range of scores is from 0 to 18.

TABLE 7 ABOUT HERE

The reliability of each scale was established in pilot studies for each nation for each of two samples: Thirteen years old and pre-university year students. Guttman scale analysis coefficients of reproductibility range from .77 to .92. For the final study, most scales are estimated to have coefficients ranging from .85 to .90 (Husén, vol. II, 114-119).

A factor analysis of the Ontario data using scores of individual students yielded three factors. The two descriptive scales formed one factor; the attitude scale that measured the degree to which pupils believe mathematics is an open process formed another; the remaining four attitudes grouped together. This would indicate that the IEA scale essentially measure three things: (1) attitudes towards mathematics as desirable, useful, enjoyable and helpful in

TABLE 6. ATTITUDES TOWARD SCHOOL AND SCHOOL LEARNING.

ITEMS
1. I generally like my school work.
2. Most school learning has little value for a person.
3. I dislike school and will leave just as soon as possible. (D)
4. I am bored most of the time in school. (D)
5. I enjoy everything about school.
6. School is not very enjoyable, but I can see value in getting a good education.
7. The most enjoyable part of my life is the time I spend in school.
8. I like all school subjects.
9. I enjoy most of my school work and want to get as much additional education as possible.
10. Although school is difficult, I want as much education as I can get.
11. I find school interesting and challenging.

TABLE 7. ATTITUDES TOWARD MAN AND HIS ENVIRONMENT.

ITEMS
1. It should be possible to eliminate war once and for all.
2. Success depends to a large part on luck and fate. (D)
3. Some day most of the mysteries of the world will be revealed by science.
4. By improving industrial and agricultural methods, poverty can be eliminated in the world.
5. With increased medical knowledge, it should be possible to lengthen the average life span to 100 years or more.
6. Someday the deserts will be converted into good farming land by the application of engineering and science.
7. Education can only help people develop their natural abilities; it cannot change people in any fundamental way. (D)
8. With hard work anyone can succeed.
9. Almost every present human problem will be solved in the future.

solving world problems; (2) belief about the nature of mathematics as an open process; and (3) views of the methods employed by teachers. (Table 8).

TABLE 8 ABOUT HERE

Purpose

The purpose of this report is to replicate a portion of the IEA study. Consequently, the hypotheses underlying the data analysis test the assumptions of relationships between attitudes and achievement, where achievement is class mean scores on the Stanford Achievement Test (SAT). Mathematics - Part A.

Findings

The average scores for this sample of students, most of whom fall within the 14-16 age group, does not appear to differ in any appreciable manner from the means for all other nations. Where differences exist, they indicate that Ontario students feel that mathematics is easier and more important than do most other groups. Ontario students also seem to enjoy school more and to have more faith in the efficacy of science (Table 9).

TABLE 9 ABOUT HERE

Attitudes and Input Variables.

Two input variables are considered here: Scholastic aptitude as measured by the School and College Ability Test (SCAT) series II forms 2A and 3A and level of education attained by the parents of the students.

TABLE 8.
FACTOR ANALYSIS OF DESCRIPTION AND ATTITUDE VARIABLES WITH 3 FACTOR VARIMAX ROTATION SOLUTION (N = 535)

VARIABLES	FACTOR LOADINGS		
	Attitudes	Views	Beliefs
1. views of teaching		.82	
2. views of learning		.72	
3. math as a process			.93
4. difficulty of math	.68		
5. importance of math	.65		
6. attitude to school	.68		
7. man and environment	.67		
Proportion of variance	.29	.18	.13

Only factor loadings >.3 are shown.

TABLE 9.
MEANS OF ATTITUDE SCALES FOR ONTARIO SAMPLE, UNITED STATES AND 10 NATION TOTAL

VARIABLES	ONTARIO SAMPLE		UNITED STATES		10 NATIONS	
			1a	3a	1a	3a
1. views of teaching	12.5	24.3	23.8	24.7	23.8	23.8
2. views of learning	11.8					
3. math as process	7.5		8.0	7.4	7.7	6.6
4. difficulty	10.9		9.4	8.5	9.1	8.2
5. importance	9.6		8.6	7.1	8.8	8.1
6. attitude toward school	11.6		8.4	8.0	9.3	9.2
7. man and environment	10.6		7.9	7.4	8.6	8.0
N	535		6231	1568	27,228	9007

Population 1a are 13 year-olds; population 3a are mathematics students in final secondary year. Data for U.S. and 10 nations are from Husén, Vol. II, ch. 4. In the I.E.A. study, scales 1 and 2 were collapsed into one scale for purposes of analysis.

Aptitude scores are associated with one descriptive variable, the view that school and school learning is oriented toward student activity rather than toward teacher-domination. Correlations between this view and mathematical aptitude is .50 ($p < .01$) and between this view and verbal aptitude is .32 ($p < .05$). Similarly the level of parents' education is associated with this view ($r = .30$, $p < .05$). These results may suggest that teachers allow brighter students more opportunity to initiate activities and to participate in discussions.

TABLE 10 ABOUT HERE

Generally, these input variables are not related to the attitudinal variables.

Attitudes and Classroom Climate

Classroom climate as measured by the Learning Environment Inventory (LEI) is highly correlated to the two descriptive measures developed by the I.E.A. The multiple R between the L.E.I. scales and each of the descriptive variables is .75 (Table 11). The most significant relationships are between the L.E.I. scales and the view about school and school learning scale. Student activity oriented classes tend to be more cohesive, more organized, more democratic and less formal than teacher-dominated classrooms. In the inquiry-centered classes pupils possess a more favorable self-concept; they like their classes; they perceive less conflict in their classes; they perceive their classrooms as adequate for their needs. The same L.E.I. variables are associated in the same manner with achievement (O'Reilly, 1975).

TABLE 10.
CORRELATION MATRIX OF CLASS MEAN SCORES FOR INPUT VARIABLES AND ATTITUDES TO MATHEMATICS AND MATHEMATICS LEARNING (N=48)

ATTITUDES	INPUT VARIABLES	
	SCAT TOTAL	PARENTS EDUCATION
1. View...teaching	.22	.00
2. View...learning	.45*	.30*
3. Math as process	.21	-.12
4. Difficulty of math	.16	-.10
5. Importance of math	.31*	.09
6. Attitude to school	.13	-.11
7. Attitude to man	.10	-.14

* $P < .05$

TABLE 11 ABOUT HERE

None of the other I.E.A. scales are significantly correlated with the classroom environment scales of the L.E.I. (O'Reilly and Garland, 1974, p. 49). These results support the classification of the first two scales, measuring the views of students, as environmental variables.

Descriptions of mathematics classes and achievement

The hypotheses to be tested are that inquiry methods of teaching are associated with higher levels of achievement in mathematics and that classes oriented toward student activity achieve higher mathematics scores than teacher-dominated classes. Although many educators take these hypotheses for granted, large scale field studies supporting these claims are rare.

The results of the I.E.A. study did not support the hypotheses. Treating the two descriptive scales as one summated scale, the I.E.A. researchers reported small positive correlations between achievement and inquiry-methods of teaching for younger students and small but negative correlations between these two variables for older students. Although some countries produced correlations larger than .08, the level required for statistical significance, most correlations were not significant. The I.E.A. data clearly does not provide support for the general hypothesis. Moreover, the possibility, generated by their data, that inquiry and student-centered methods of teaching are dysfunctional for older students should not go untested. The Ontario data is not from terminal high school students or 18 years old. However it does show that student activity oriented classes have higher achievement than teacher-

TABLE 11.
CORRELATIONS BETWEEN LEARNING ENVIRONMENT INVENTORY SCALES AND I.E.A. DESCRIPTIVE SCALES.

ENVIRONMENT SCALES	DESCRIPTIVE SCALES	
	VIEWS ABOUT MATH. TEACHING	VIEWS ABOUT SCHOOL LEARNING
1. Cohesiveness	-.02	.36*
2. Diversity	.16	-.10
3. Formality	.15	-.37*
4. Speed	-.24	-.08
5. Environment	.24	.37*
6. Friction	-.42*	-.30*
7. Goal Direction	.07	.14
8. Favoritism	-.07	-.33*
9. Cliqueness	.09	-.17
10. Satisfaction	.22	.35*
11. Disorganization	-.25	-.33*
12. Difficulty	-.10	.17
13. Apathy	-.16	-.40*
14. Democratic	.09	.48*
15. Competitiveness	.12	.22
R.	.75*	.75*

*$p < .05$

dominated classes (r = .56). A good description of these classes was seen in the comparison of the L.E.I. scales and the descriptive scale views about school and school learning.

TABLE 12 ABOUT HERE

An argument can be made that teachers make a priori assumptions about their classes and allow students more or less initiative depending on the student's social and ability characteristics. Note that the descriptive scale which correlates positively with achievement across all population, views of school learning, is also related to level of parents' education and to pupil aptitude. Further, although the means of the two descriptive variables are similar for grades nine and ten of the Ontario sample, the means for the advanced course classes are higher than for the general course classes.

Since the correlations between the descriptive scale views about mathematics teaching and achievement tend to be non-significant, any interpretations must be viewed as highly speculative. It does appear than when one deals with a class of lesser ability, (I.E.A. population 3a and Ontario general level program students), or when older students must be prepared for a rigid examination system (I.E.A. populations 3a and 3b), mechanistic, rate methods of teaching may be more effective. One possible reason for this in the case of less competent students, may be that they have not developed a stock of suitable heuristics to do well with inquiry methods; and at the examination level, teachers become adept at teaching to the exam. It should be noted that at the grade 9 and 10 levels in Ontario, pupils are not subjected to any rigid examination system.

TABLE 12.
CORRELATIONS BETWEEN MATHEMATICS ACHIEVEMENT AND INQUIRY METHODS OF TEACHING FOR I.E.A. AND ONTARIO SAMPLES

SAMPLE	GROUPS			
I.E.A. WITHIN NATIONS	1a	1b	3a	3b
Summated descriptive scale[1]	.10*	.05	-.03	-.07
I.E.A. BETWEEN NATIONS (N = 10)				
View of math teaching	.70*	.15	-.51	-.59
Views of school learning	.60	.23	.10	.56

ONTARIO SAMPLE	GENERAL (N=16)	ADVANCED (N=32)	TOTAL N=48
View of math teaching	-.33	.12	.20
View of school learning	.09	.60**	.56**

[1] The correlacions shown are approximate median correlations taken from Husén, vol. II, figure 4.1, p. 149. *$p < .05$; **$p < .01$.

The fact that achievement of pupils consistently related the views about teaching in both the Ontario and I.E.A. studies may be due to the lack of the discrimating power of the scale itself. An examination of the items indicate that most items do not directly relate to the use of inquiry techniques and the use of heuristics. Most items seem to refer only to limited motions of mastering problem solving, rather than to wides aspects of discovery in all phases of learning mathematics.

As a conclusion, it appears that the less formal, student activity oriented classes achieve better than teacher-dominated classes. It is suggested that the teacher behaviors reflected by these descriptive measures are functions of pupil characteristics.

Attitudes and achievement

Attitudes towards a subject should influence the amount of effort expended in learning a subject, and thus influence achievement.

On the basis of the I.E.A. data it was concluded that affective outcomes of teaching are not consistently related to cognitive outcomes. In this study, the four attitudes cluster together in one factor, but the only one which is related to achievement is the attitude that mathematics is important to society ($r = .30$). This finding is supported by the I.E.A. data between national mean scores for these two variables, which yield correlations between .27 to .88. The larger correlations result from the samples of older students (Husén, vol. II, 156).

TABLE 13 ABOUT HERE

TABLE 13.
CORRELATIONS BETWEEN MATHEMATICS ACHIEVEMENT
AND ATTITUDES OVER ALL NATIONS AND FOR ONTARIO

VARIABLES	IEA POPULATIONS				ONTARIO
	1a	1b	3a	3b	
Difficulty of math	.00	-.02	-.07	.00	.08
Importance of math	.04	.08	.09	.13*	.30*
School learning	-.16*	-.06	.57*	.65*	-.01
Man and environment	-.18*		.69*	.49*	.02

* $p < .05$

Although low correlations were reported by Husén for the variables achievement and difficulties of learning mathematics for individuals, correlations between national means were negative for the four populations. The range of correlations was from -.23 to -.64 (Husén, vol. II, 156). In the Ontario study no such correlation was found for any sub-sample. However, the correlation between the L.E.I. scale "difficulty" and achievement was .33 ($p < .05$) (O'Reilly, 1975). High scores on this scale indicate that the classwork is difficult. These data permit the tentative conclusion that mathematics achievement is associated with the attitude that mathematics is difficult.

The attitude variable school and school learning is essentially an index of liking for school and school work. The I.E.A. data show positive correlations between this variable and achievement, especially for older students, but not for the Ontario sample. However, in the Ontario study, the L.E.I. variable, satisfaction, also an indicator of student liking for their class, is positively related to achievement ($r = .45$) (O'Reilly, 1975). A probable conclusion is that achievement in mathematics is associated with an attitude of liking for school and school work.

The attitude that man can control his social and physical environment through the use of science and knowledge is related to achievement in mathematics in the I.E.A. study but not in the Ontario study.

Achievement and belief about the nature of mathematics

The factor analysis of attitudes identified the variable attitudes toward mathematics as a process as somewhat distinct from the other attitude scales employed by the I.E.A. This variable is singled out,

for the main purpose of the new mathematics movement was not so much to alter teaching methods but to alter the image students have of the structure and process of mathematics. If these programs give students a truer image of mathematics, then that is good.

The correlation for the class means between this belief and achievement for the 32 advanced classes for students who may be preparing themselves for college-level study of mathematics is significant ($r = .47$). These students would have been most consistently exposed to new mathematics curricula. The general course students, whose achievement scores are much lower than those of the advanced students, perhaps have had teachers who adopted more mechanistic methods of teaching and stressed "fundamentals" rather than permitting student initiative to explore ways in which mathematics could be considered an open system. Another possible explanation is that advanced level students, and more capable students are better able to benefit from the theory of mathematics as an open system.

How is the belief that mathematics is an open system associated with teaching methods? Since this variable is not significantly related to the input variables reported above, it can be assumed that teachers directly influence the students' beliefs about the nature of mathematics. The I.E.A. report shows consistently negative correlations between achievement and the belief that mathematics is an open process. Since, at the time of the I.E.A. study, new mathematics curricula were relatively new and were not wide-spread, the findings were not surprising. Previous mathematics learning, texts, methods and exams were geared to a more traditional view of mathematics. Those students who did hold that mathematics is an open system would have been out-of-phase with the entire system.

On the other hand, the Ontario sample can be presumed to have been in a system in which the official curriculum guide-lines had prescribed new mathematics curricula since these students first enrolled in the public school system. The students were initiated in the study of mathematics with the concepts, methods and principles of new mathematics curricula. Thus the results for these students are expected to be quite different.

TABLE 14 ABOUT HERE

Table 14 shows the results. Although the correlation between the Ontario class means is positive, it fails to reach the level required for significance. However, for the I.E.A. groups, correlations for individual scores and national mean scores are consistently negative. We can tentatively conclude that in new mathematics programs, unhampered by rigid testing programs, the belief that mathematics is an open system is associated with high achievement.

Regression Analysis

A stepwise regression analysis was computed to assess the relationships between views, attitudes and beliefs taken together and achievement. Since two of the attitude variables are less successfully related to achievement than are their counterparts in the Learning Environment Inventory, the L.E.I. scales "difficulty" and "satisfaction" were substituted for the attitude scales difficulty and school and school learning. The most important variables in the regression equation, in order of importance, are:

TABLE 14.
CORRELATIONS BETWEEN NATIONAL MEAN SCORES AND INDIVIDUAL SCORES FROM THE I.E.A. STUDY AND CLASS SCORES OF THE ONTARIO STUDY FOR THE VARIABLES MATHEMATICS ACHIEVEMENT AND BELIEF ABOUT THE NATURE OF MATHEMATICS

			POPULATIONS			
	ONTARIO		OVER ALL NATIONS		NATIONAL MEANS	
TOTAL	GENERAL	ADVANCED	1a	3a	1a	3a
.26	-.01	.47*	-.08	-.24*	-.78*	-.54

* $p < .05$

views of school learning, importance of mathematics, difficulty (LEI), satisfaction (LEI), and man and environment. The total R-SQUARED value is .55.

TABLE 15 ABOUT HERE

Thus the classroom environment, including teacher behavior, is considerably more important to achievement in mathematics than are student attitudes and beliefs about the nature of mathematics. Belief about the nature of mathematics as an open process is not related to achievement.

Discussion

The results of this study contrast in specific ways the results of the I.E.A. study. Although the Ontario sample is not a national one, it includes considerable variety and can be considered as representative of classes in which most of the 14 and 15 years old students in Ontario are located. Since the I.E.A. study populations comprised for the most part thirteen years old students, and those eighteen years old who were still in school, the results of the two studies are not directly comparable. However, this study does provide some direction to those who are skeptical of some of the findings of the I.E.A. study and who may wish to design studies to replicate portions of it which relate to attitudes.

Husén, in his analysis, suggested that inquiry methods of learning are functional for younger secondary students, but dysfunctional for older students, especially those faced with rigid examinations.

TABLE 15.
STEPWISE REGRESSION ANALYSIS OF ATTITUDE, BELIEF AND VIEW VARIABLES ON MEAN CLASS ACHIEVEMENT IN MATHEMATICS (N = 48)

VARIABLES	r	R	RSQ	RSQ-change
Views of school learning	.56	.56	.314	
Importance of mathematics	.30	.68	.460	.146
Difficulty of class (LEI)	.33	.71	.503	.043
Satisfaction (LEI)	.45	.73	.531	.028
Man and environment	.02	.74	.545	.014

The Ontario data suggest that for grades 9 and 10 level students, inquiry methods do lead to higher achievement, particularly in the case of students in advanced classes. Only in the case of general program students, where mathematics courses tend to reduce the theoretical content in favor of the more practical implications of arithmetic, does the mechanistic, drill method of teaching appear to be superior. However, even in this case, the statistical analysis is not significant. In interpreting these data, it should be pointed out that little hard evidence is available. The Husén data presents only two significant correlations, both for populations 1a (all thirteen years old). Why similar results do not obtain for populations 1b (classes in which most students were thirteen years old) is not explained. Perhaps the 1a includes students who were unable to keep up with their age cohorts. For practical purposes, it may be suitable to consider all of the Husén data as non-significant in the statistical sense. For the Ontario data, the <u>view of school and school learning</u> as student activity-centered is related to achievement, both for the total sample and the sample of 32 advanced level classes. The dissimilarity of results for the general level classes suggests some interpretations related to the literature on heuristic methods of learning (Hughes, 1974), Bruner's analysis of student learning (Bruner and Kenney, 1973) and the literature on creativity and mathematical invention (Gordon, 1968; Hadamard, 1945). In the opinion of each of these authors, discovery learning is the result of highly idiosyncratic cognitive behavior in which sensory impressions and analogies assist the individual to re-organize and

re-view problems so that discoveries are made and previously learned material is retained. Less capable students may not be given the opportunity for such activity or they may be less capable of profiting from it. Since there is little training available to teachers in heuristic methods, it may be that teachers are unable to help students acquire a stock of suitable heuristics.

The data on attitudes presents support for the hypothesized relationships between achievement and the attitude that mathematics is important but not for other attitudes. The conclusion that mathematics achievement is related to the attitude that mathematics is difficult is contradictory to the general hypothesis. Should this association be up-held in other studies, mathematics teachers will continue to be faced with students who feel that only a few can be successful in mathematics. Policy-makers will tend to reduce the role of mathematics in educational programs to providing only certain fundamentals of arithmetic to all students. The following remark by Sawyer may be quite appropiate:

> Many people regard mathematicians as a race apart, possessed of almost super-natural powers. While this is very flattering for successful mathematicians, it is very bad for those who...are attempting to learn the subject (Sawyer, 1943:7).

Further research on attitudes toward mathematics would assist educators with this problem.

References

Aiken, Lewis R., "Attitudes Toward Mathematics", Review of Educational Research, 1970, 40, 551-596.

Anderson, Gary J. and Walberg, H.J., "Learning environments", in H.J. Walberg (Ed.). Evaluating Educational Performance, Berkeley, Cal.: McCutchan Publishing, 1974.

Anderson, Gary J., "Effects of Classroom Social Climate on Individual Learning", American Education Research Journal, 1970, 7, 135-152.

Anderson, Gary J. and Walberg, H.J., "Classroom Climate and Group Learning", International Journal of the Educational Sciences, 1968, 2, 175-180.

Anttonen, R.G., An Examination into the Stability of Mathematics Attitude and its relationship to mathematic achievement from elementary school to secondary school level. (Doctoral dissertation, University of Michigan, 1967) Ann Arbor, Mich.: University Microfilms, 1968, No. 68-1521.

Bruner, Jerome S. and Kenney, Helen J., "Representation and mathematics learning", in J.M. Anglin, (Ed.), Jerome S. Bruner: Beyond the information given, New York: W.W. Norton, 1973.

Dutton, Wibbur H., "Attitudes of Junior High School Pupils Toward Arithmetic", School Review, 1956, 64, 18-22.

--------, "Another Look at Attitudes of Junior High School Pupils Toward Arithmetic", Elementary School Journal, 1968, 68, 265-8.

Gordon, William J.J., Synectics: The development of creative capacity. New York, Collier, 1961.

Hadamard, Jacques. The psychology of invention the mathematical field. Princeton, Princeton University Press, 1954.

Henderson, Kenneth B., "Research on Teaching Secondary School Mathematics", in N.L. Gage, (Ed.). Handbook of Research on Teaching, Chicago: Rand McNally, 1963.

Hughes, Barnabas B., "Heuristic Teaching in Mathematics", Educational Studies in Mathematics, 1974, 5, 291-299.

Husén, Torsten (Ed.) International Study of Achievement in Mathematics: A Comparison of Twelve Countries, New York: John Wiley, two volumes, 1967.

Neale, Daniel C., The role of attitudes in learning mathematics", Arithmetic Teacher, 1969, 16, 631-640.

O'Reilly, Robert R. and Garland, Parnell, Paradigm for Evaluation in the High School, Ottawa: University of Ottawa, 1974.

O'Reilly, Robert R., "Classroom climate and achievement in secondary school mathematics classes", Alberta Journal of Educational Research, 21, 241-8, 1975.

Rombey, T.A., "Current Research in Mathematics Education", Review of Educational Research, 1969, 39, 473-492.

Sawyer, W.W., Mathematician's Delight, London: Pelican Books, 1943.